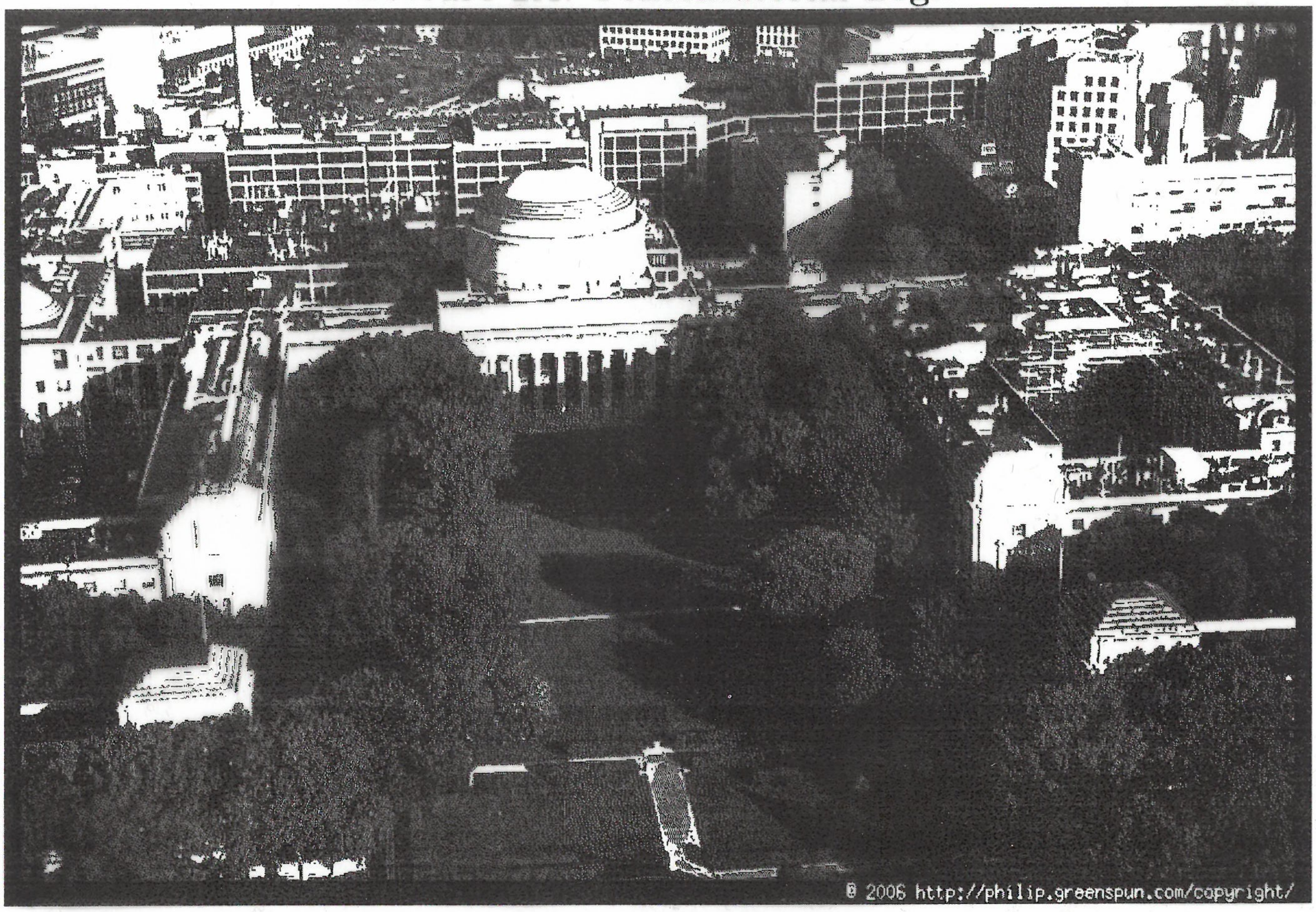
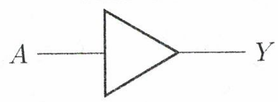
Build a Digital Clock from the 1980's

Lecture 2A: Combinatorial Logic



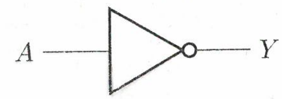
Original photo courtesy of Philip Greenspun (`http://philip.greenspun.com/`)
Edited using the GNU Image Manipulation Program (`https://www.gimp.org/`)

YES Gate



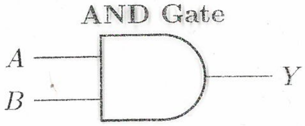
A	Y
0	0
1	1

NOT Gate



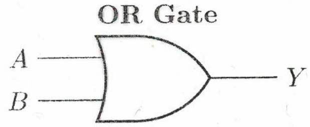
A	Y
0	1
1	0

AND Gate



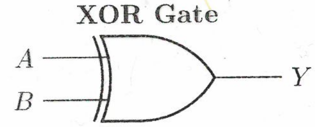
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate



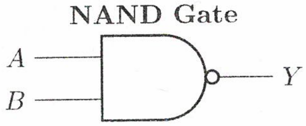
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

XOR Gate



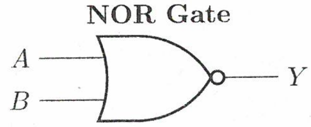
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

NAND Gate



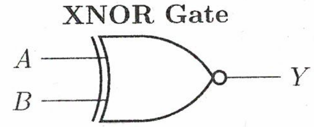
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NOR Gate



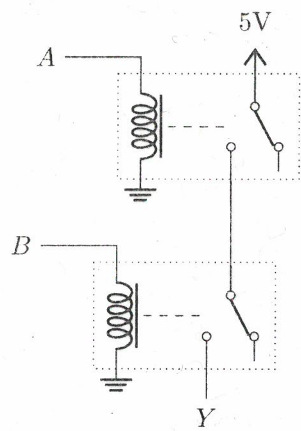
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

XNOR Gate

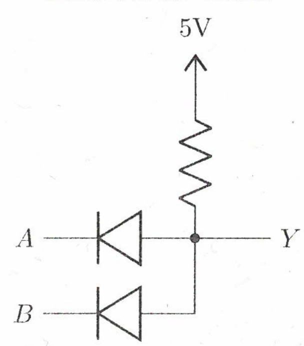


A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

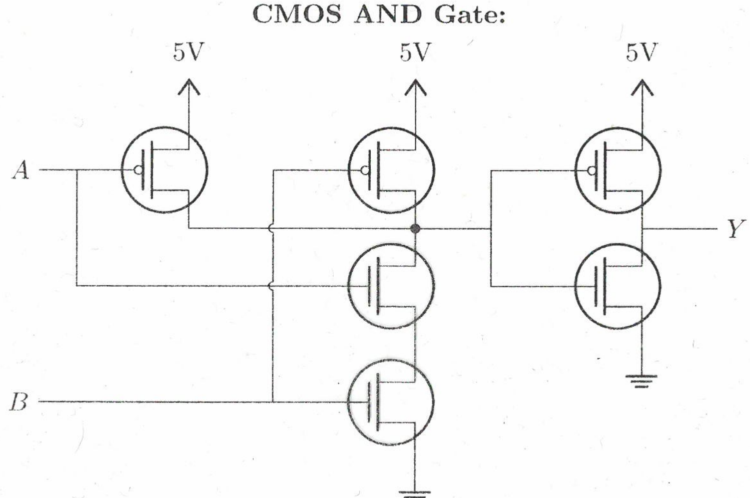
Relay AND Gate:



DRL AND Gate:



CMOS AND Gate:



Numerical Representations

Decimal:

6873

$$(6 \times 10^3) + (8 \times 10^2) + (7 \times 10^1) + (3 \times 10^0)$$

Sexagesimal:

$$(1 \times 60^2) + (54 \times 60^1) + (33 \times 60^0)$$
$$(1 \times 3600) + (54 \times 60) + (33 \times 1)$$
$$3600 + 3240 + 33 = 6873$$

Binary:

1101011011001

$$(1 \times 2^{12}) + (1 \times 2^{11}) + (1 \times 2^9) + (1 \times 2^7) + (1 \times 2^6) + (1 \times 2^4) + (1 \times 2^3) + (1 \times 2^0)$$
$$4096 + 2048 + 512 + 128 + 64 + 16 + 8 + 1$$

$$\begin{array}{r} 1 \\ 172 \\ +\;253 \\ \hline 425 \end{array}$$

$$\begin{array}{rccccc} & 1 & 1 & 1 & & \\ & 0 & 1 & 1 & 1 & 0 \\ + & 0 & 1 & 0 & 1 & 1 \\ \hline & 1 & 1 & 0 & 0 & 1 \\ & 16 & 8 & 4 & 2 & 1 \end{array}$$

$\boxed{14}$

$\boxed{11}$

$\boxed{25}$

10

MSB

$a \backslash b$	0	1
0	0	0
1	0	1

ab

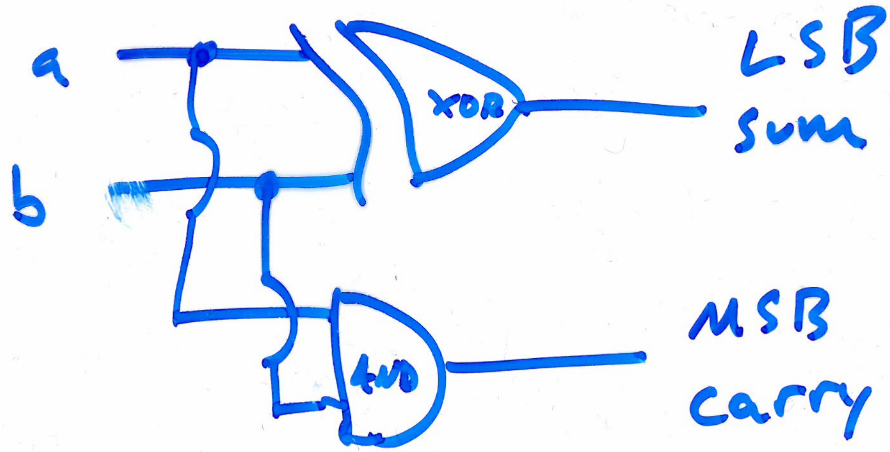
LSB

$a \backslash b$	0	1
0	0	1
1	1	0

$a \oplus b$

$a+b$

"half-adder"



MSB "carry"

c \ a, b	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$\boxed{bc + ab + ac}$$

3 AND, 2 OR

a	b	c	c	s
0	0	0	0	0
0	1	0	0	1
1	1	0	1	0
1	0	0	0	1
0	0	1	0	1
0	1	1	1	0
1	1	1	1	1
1	0	1	1	0

$$ab + c(b+a)$$

$$ab + c((a\oplus b) + ab)$$

$$ab + c(a\oplus b) + \cancel{ab}$$

$$ab + c(a\oplus b)$$

LSB "sum"

c \ a, b	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$\boxed{c\bar{a}\bar{b} + b\bar{a}\bar{c} + a\bar{b}\bar{c} + abc}$$

8 ANDs + 3 OR's + ~~4~~ 3 NOT's

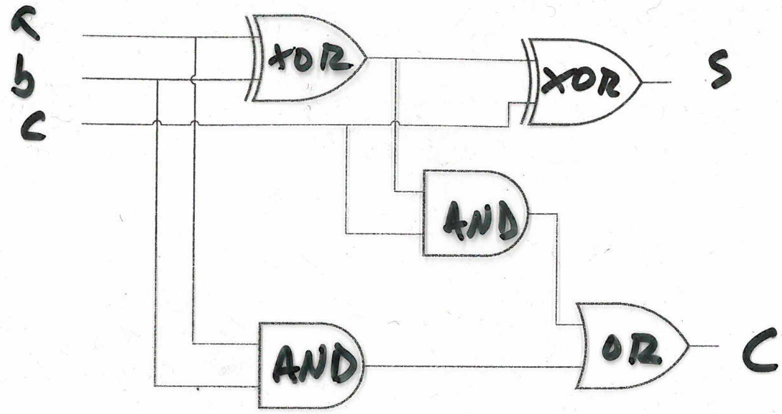
14

$$c(ab + \bar{a}\bar{b}) + \bar{c}(\bar{a}b + \bar{b}a)$$

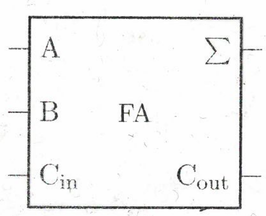
$$\overline{a\oplus b}$$

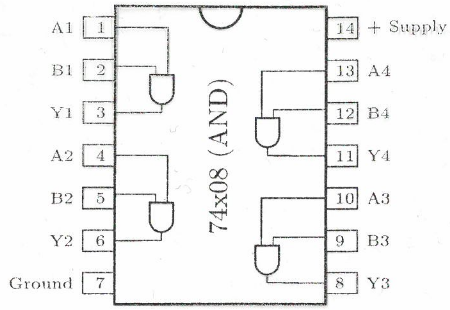
$$c(\overline{a\oplus b}) \qquad \bar{c}(a\oplus b)$$

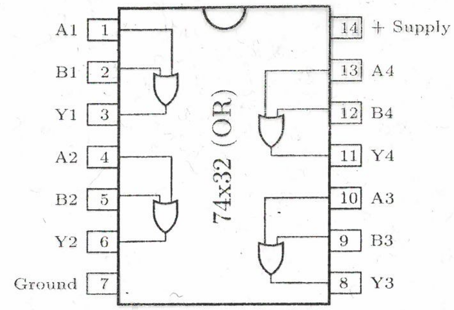
$$(a\oplus b)\oplus c$$

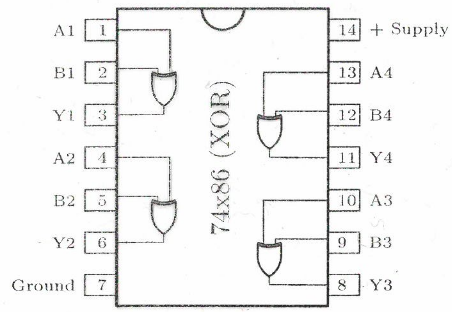


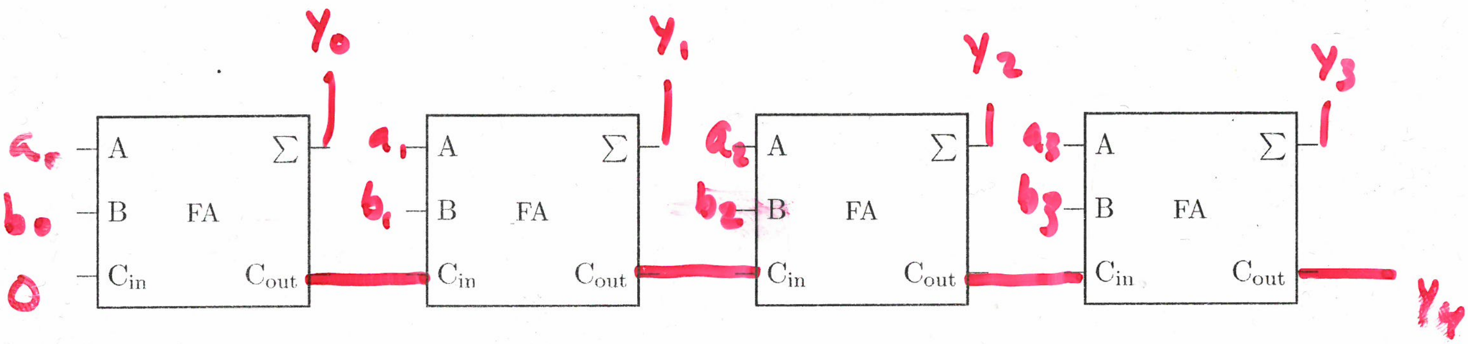
"full adder"




74x08 (AND)
A1
B1
Y1
A2
B2
Y2
Ground
+ Supply
A4
B4
Y4
A3
B3
Y3


74x32 (OR)
A1
B1
Y1
A2
B2
Y2
Ground
+ Supply
A4
B4
Y4
A3
B3
Y3


74x86 (XOR)
A1
B1
Y1
A2
B2
Y2
Ground
+ Supply
A4
B4
Y4
A3
B3
Y3



add a+b

0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

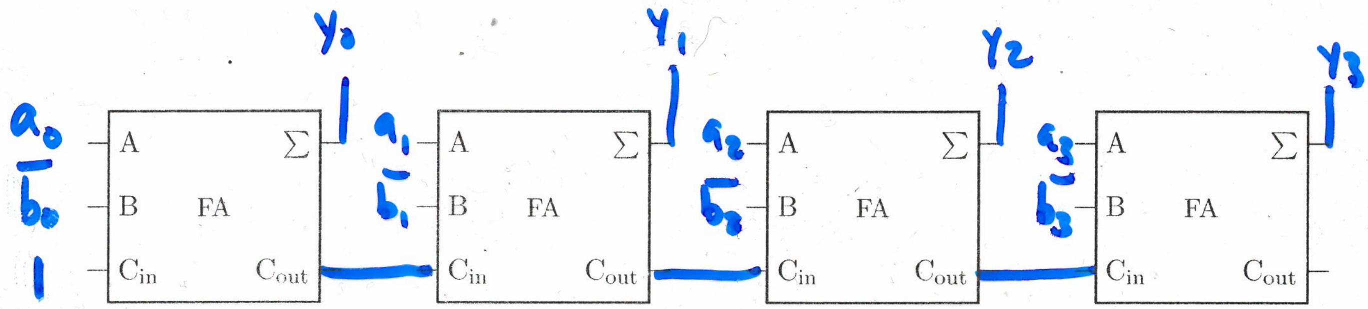
$4 \to -4$

$0100 \to 1100$

$$\boxed{2^N - x}$$

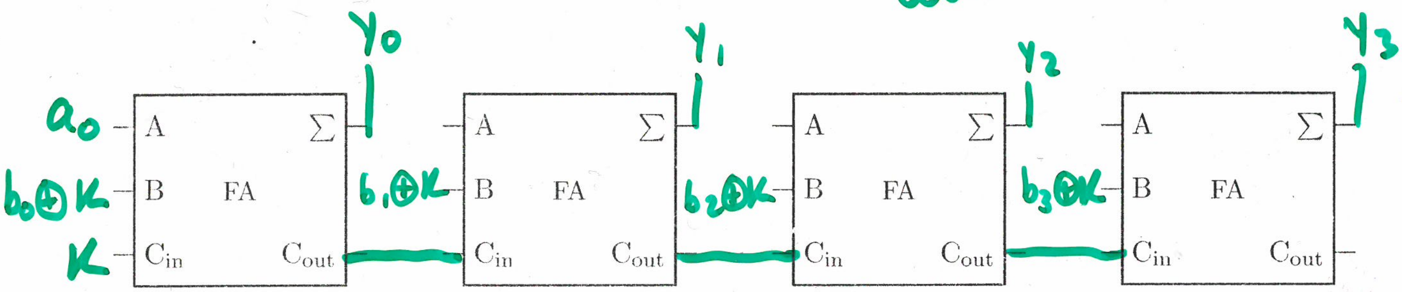
$$\underbrace{2^N - 1 - x} + 1$$

$\sim x + 1$


y_0
y_1
y_2
y_3
a_0
$\overline{b_0}$
1
a_1
$\overline{b_1}$
a_2
$\overline{b_2}$
a_3
$\overline{b_3}$
A
B
C_{in}
Σ
FA
C_{out}
A
B
C_{in}
Σ
FA
C_{out}
A
B
C_{in}
Σ
FA
C_{out}
A
B
C_{in}
Σ
FA
C_{out}

subtract a-b

K is a separate input: subtract if 1, add if 0



$y = a+b$ if $K=1$ else $a-b$