Part I

Here is a truth table.
Start with Sum of Products (S.O.P.)
and simplify it

a	b	c
0	0	0
0	1	1
1	0	1
1	1	1

yes this is just OR... but let's make sure Boolean Algebra can help us reduce to simplified result. should reduce to c=a+b

S.O.P.:

$c = \bar{a}b + ab + a\bar{b}$

hmmm $c = \bar{a}b + a(\bar{b}+b)$

$= \bar{a}b + a(1)$

$= \bar{a}b + a$

... hmmm what if instead...

$c = \bar{a}b + ab + a\bar{b}$

$(\bar{a}+a)b + a\bar{b}$

$c = (1)b + a\bar{b}$

$= b + a\bar{b}$

... hmmm W.T.F.?

WE MUST USE IDEMPOTENCE

$a = a + a$ or $a = a \cdot a$

so retry.

$c = \bar{a}b + ab + a\bar{b}$

mitosis

$= \bar{a}b + ab + ab + a\bar{b}$

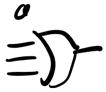
$b(\bar{a}+a) + a(b+\bar{b})$

$\boxed{c = b + a}$ noice

COULD HAVE ALSO GENERATED A TRUTH TABLE

a	b	c	y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Keep this in mind since it does quickly indicate to us this is a



1960s EE boss. Could smoke in office then

Build a circuit with output y and inputs a, b, c.
y should be high if a is on or if b is on with c off or if c is on or if b is off with a on.

Just written out logic

$y = a + b\bar{c} + c + \bar{b}a$

then $y = a(1+\bar{b}) + b\bar{c} + c$

$y = a + b\bar{c} + c$

$y = a + b\bar{c} + c = b + c$

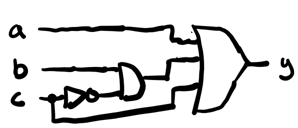
$= a + b\bar{c} + cb + c\bar{b} + c$

$= a + b(\bar{c}+c) + c(1+\bar{b})$

$= a + b(1) + c(1)$

$y = a + b + c$

Build circuit



IS THERE A BETTER / MORE RELIABLE WAY TO REDUCE??

YES. KARNAUGH MAPS!!

KARNAUGH MAP

ASIDE... HIGHER DIMENSIONAL TRUTH TABLE

OR

a	b	c
0	0	0
0	1	1
1	0	1
1	1	1

KMAP:

b \ a	0	1
0	0	1
1	1	1

a

b

$c = a + b$

As applied to our $y = a + b\bar{c} + c + \bar{b}a$

c \ ab	00	01	11	10
0	0	1	1	1
1	1	1	1	1

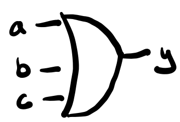
b

a

c

so $\therefore$ $y = a + b + c$

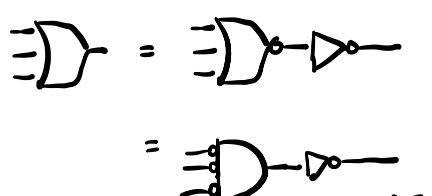
so



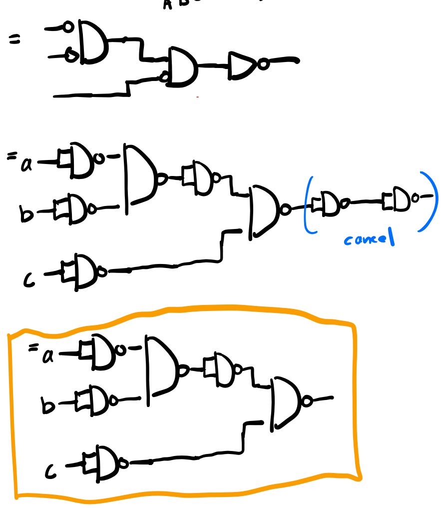
great!

BUT WHAT IF?

NEED deMorgan

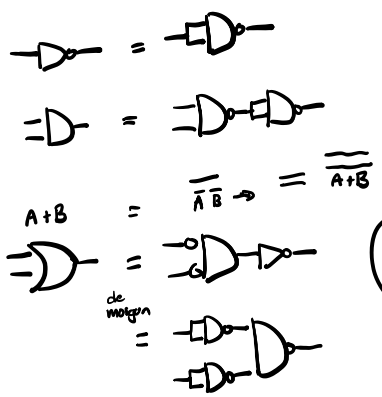


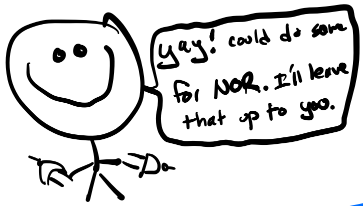
$\bar{A}\bar{B}\bar{C} = (\bar{A}\bar{B})\bar{C}$



Stuff like this happened a lot. or sometimes you'd have left over gates so you'd try to use those gates to avoid bringing in more chips to the design

All logic can be represented as sum of products. (S.O.P.)
S.O.P. has + · $\overline{\ }$
make those from you're good





Let's build some more legit logic...

A = 2 bits   B = 2 bits

$$\boxed{Y = A > B}$$

A_1	A_0	B_1	B_0	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$$\boxed{A_1\bar{A_0}\bar{B_1}}(\underbrace{\bar{B_0} \cancel{+ B_0}}_{1})$$

$$Y = \bar{A_1}A_0\bar{B_1}\bar{B_0} + (A_1\bar{A_0}\bar{B_1}\bar{B_0} + A_1\bar{A_0}\bar{B_1}B_0)$$

$$(+ A_1A_0\bar{B_1}\bar{B_0} + (A_1A_0\bar{B_1}B_0) + A_1A_0B_1\bar{B_0})$$

$$A_1A_0(\bar{B_1}\bar{B_0} + \bar{B_1}B_0) \qquad A_1A_0\bar{B_0}(B_1 \cancel{+ \bar{B_1}})$$

$$A_1A_0(\bar{B_1}(\underbrace{\bar{B_0} + B_0}_{1}))$$

$$\boxed{A_1A_0\bar{B_1}}$$

$$Y = \bar{A_1}A_0\bar{B_1}\bar{B_0} + A_1\bar{A_0}\bar{B_1} + A_1A_0\bar{B_1} + A_1A_0B_1\bar{B_0}$$

Idempotent

$$A_1\bar{B_1}$$

$$A_0\bar{B_0}(\bar{A_1}\bar{B_1} + A_1B_1)$$

$$A_0\bar{B_1}\bar{B_0}(\bar{A_1})$$

$$A_1A_0B_1\bar{B_0} + A_1\bar{B_1}$$

KARNAUGH MAP

B_1B_0 \ A_1A_0	00	01	11	10
00	0	1	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	1	0

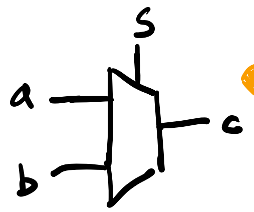
$A_0\bar{B_1}\bar{B_0}$

$A_1\bar{B_1}$

$A_1A_0\bar{B_0}$

NOT SURE!!!

Now MAKE MULTIPLEXER



if (s) $c=a$

else $c=b$

Like a solid state SPDT RELAY

STEP 1: TRUTH TABLE

a	b	s	c
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

KMAP:

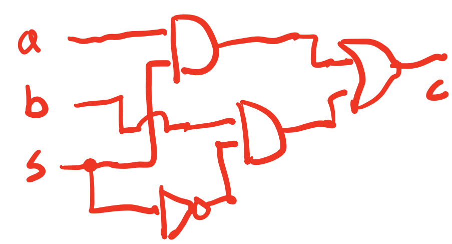
s \ ab	00	01	11	10
0	0	1	1	0
1	0	0	1	1

$b\bar{s}$

as

just two for full coverage

$c = as + b\bar{s}$



$c = \bar{a}b\bar{s} + a\bar{b}s + ab\bar{s} + abs$

sum of products

could simplify from here if we wanted.